Dual And Quad Flag And Loop Array Phaser Theory

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Any phased array has loss (or in some cases gain) due to the phase difference of the signals from two antennas which are combined to produce the nulls. This loss (or gain) depends on (1) the separation of the two antennas, (2) the arrival angle of the signal, (3) the method used to phase the two antennas, and (4) the types of antennas. Let $\varphi + 180°$ be the phase difference for a signal arriving at the two antennas. It can be shown by integrating the difference of the squares of the respective cosine functions that the amplitude A of the RMS voltage output of the combiner given RMS inputs with RMS amplitudes $E(\theta)$ is equal to to $E(\theta)\sqrt{(1 - \mathrm{COS}(\varphi))}$ where $E(\theta)$ is the amplitude of a ray whose vertical plane makes an angle θ with the plane of the array where all antenna elements lie in a plane passing through the center of the earth. In other words,

$$A = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} 2\,E^2(\theta)(\cos(x) - \cos(x+\varphi))^2\,dx} = E(\theta)\sqrt{1-\cos(\varphi)}$$

The dB gain or loss for the combiner due to their phase difference is $20 \log\left[\sqrt{1 - \mathrm{COS}(\varphi)}\right]$.

Note that the formulas above do not calculate the pattern of the array, just the phaser loss.

Let us consider the best case, when the signal arrives from the maximum direction. For a spacing s between the centers of the two flags or loops, if $\theta = 0$ and the arrival angle is α, then the distance d which determines the phase difference between the two signals is $d = s\,\mathrm{COS}(\alpha)$. If s is given in feet, then the conversion of d to meters is $d = s\,\mathrm{COS}(\alpha)/3.28$. The reciprocal of the velocity of light $1/2.99\text{x}10^{\wedge}8 = 3.34$ nS/meter is the time delay per meter of light (or radio waves) in air. So the phase difference of the two signals above in terms of time is $T = 3.34\, s\,\mathrm{COS}(\alpha)/3.28$ nS when s is in meters. The phase difference is thus $\varphi = 0.36Tf = 0.367fs\mathrm{COS}(\alpha)$ degrees where f is the frequency of the signals in MHz and s is the distance between the centers of the two flags in feet. For 100 foot spacing and 30° arrival $\varphi = 31.8°$, which we will round of to 32° to simplify the discussion. If two flags or two loops separated by 100 feet introduce –212° delay in one direction of the plane of the flags or loops to generate a null for 30°arrival in that direction at 1 MHz, then for a 30°arrival in the opposite direction (the desired direction), the phase difference is 64°, so the loss is –2.5 dB. At 600 kHz $\varphi = 0.6 \times 31.8° = 19°$, so the phase difference is 38°, and the loss is –6.7 dB. At 1.5 MHz the loss is +0.4 dB (gain).

For a quad array with 100' spacing, assume that the 1st and 2nd pair each have –212° delay in one direction of the plane of the flags or loops to generate a null for 30°arrival in that direction. Thus each pair has the losses calculated above. Suppose that the pairs have "standard" phasing, namely the same –212° delay in one direction at 1 MHz. However, the signal delay between the two pairs is 64° at 1 MHz, so the total delay for calculating phaser loss is 96° at 1 MHz. The loss is +0.9 dB (gain). At 600 kHz the loss is –3.4 dB, and at 1.5 MHz the loss is +5.4 dB (gain). Thus the quad combiner losses and gain are –10.1, –1.6, and +5.8 dB for 600, 1000, and 1500 kHz respectively. If magic T combiners are used, then 6 dB additional loss results, for a net loss of –16.1, –7.6, and –0.2 dB for 600, 1000, and 1500 kHz respectively.

If high Z J310 – J271 active flag and loop amplifiers are used at the loop or flag elements, then the output voltages at each active amplifier vary inversely with wavelength, by about a factor of 3, or 10 dB "loss" from the high end of the MW band to the low end of the MW band. This gives the active standard QDFA its characteristic 25 dB roll off from high to low MW band.

The results of this article are not restricted to flag and loop arrays, or to dual arrays and "standard" quad arrays alone.